

If you're reading this, please contribute!

REMINDER! This is a template! The cheat sheet maintainer (.json) *intentionally* leaves extra space for you to add your own notes! If something's missing, add it yourself! (and if it's important enough please contribute!)

FSP

Syntax

```
// instance prefixing - a:P
switch = (on -> off -> SWITCH).
|| TWO_SWITCH = (a:SWITCH || b:SWITCH).

// relabeling - /(new1/old1, new2/old2, ...).
CLIENT = (call -> wait -> continue -> CLIENT).
SERVER = (request -> service -> reply -> SERVER).
|| CLIENT_SERVER = (CLIENT || SERVER) / (call/request,
    <-> reply/wait).

// process prefixing (mutex) - {a1, ..., ax} :: P
RESOURCE = (acquire -> release -> RESOURCE).
USER = (acquire -> user -> release -> USER).
|| RESOURCE_SHARE = (a:USER || b:USER || {a, b} ::
    <-> RESOURCE).
// RESOURCE is a single shared instance between
// the two USERS
// for loop syntax
|| SWITCHES (N = 3) = (forall [i:1..N] s[i]:SWITCH
    <-> )
// or alternatively
range Seats = 1..3
|| SEATS=(seat[i:1..3]:SEAT).

// hiding operator - \{a1, ..., ax}
// interface operator - @{a1, ..., ax}
// these two do the same thing
USER = (acquire -> use -> release -> USER) \ {use}.
USER = (acquire -> use -> release -> USER) @ {
    <-> acquire, release}.

// syntax for progress
progress P = {a1, ..., an}
// syntax for high priority
|| C = (P || Q) << {a1, ..., an}.
// syntax for low priority
|| C = (P || Q) >> {a1, ..., an}.

// simulating a boolean
const False = 0
const True = 1
range Bool = 0..1
```

Maker-user example

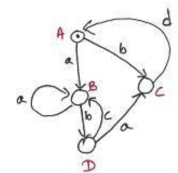
```
MAKER = (make -> ready -> MAKER).
USER = (ready -> user -> USER).
|| MAKER_USER = (MAKER || USER).
```

Garden example (maybe move this to a diff section later)

```
const N = 4
range T = 0..N
set VarAlpha = {value.(read[T],write[T])}
VAR = VAR[0],
VAR[u:T] = (read[u] -> VAR[u]
    | write[v:T]->VAR[v]).
TURNSTILE = (go -> RUN),
RUN = (arrive-> INCREMENT
    | end -> TURNSTILE),
INCREMENT = (value.read[x:T]
    -> value.write[x+1]->RUN)
+VarAlpha.

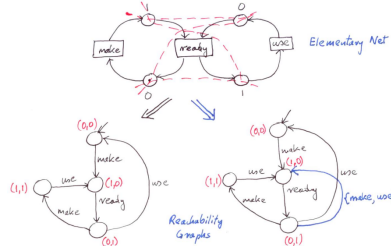
DISPLAY = (value.read[T]->DISPLAY)+(value.write[T]
    <-> ).
|| GARDEN = (east:TURNSTILE || west:TURNSTILE ||
    <-> display:DISPLAY
    || {east,west,display}::value:VAR)
/ (go /{east,west}.go,
    end/{east,west}.end).
```

LTS



$A = (a \rightarrow B | b \rightarrow C)$
 $B = (a \rightarrow B | b \rightarrow D)$
 $C = (d \rightarrow A)$
 $D = (a \rightarrow C | c \rightarrow B)$

Petri Nets Reachability Graphs



Hiding/Labeling

relabeling:
(PROCESS) / {newlabel1/oldlabel1, ..., newlabeln/oldlabeln}
interface: (PROCESS) @ {a1...ax} hides all actions except a1...ax
hiding: (PROCESS) \ {a1...an} hides actions a1...an
{a1, ..., ax} :: P replaces every action label n in the alphabet of P with the labels a1.n, ..., ax.n. Thus, every transition (n -> X) in the definition of P is replaced with the transitions ({a1.n, ..., ax.n} -> X)

Bisimulation

State Bisimilarity - $p \approx q$ iff whatever action executed at p can also be executed at q , and vice versa.
LTS Bisimilarity - $P \approx Q$ iff each state P_t , reachable from the initial state by a trace t in P is bisimilar to an appropriate state q_t that is reachable from the initial state by the same trace t in Q .

Mutual Exclusion

Arbitrary interleaving of read and write actions leads to **interference**. Interference bugs are difficult to locate. We use **mutual exclusion** to only give one process access to the shared resource at a time.

```
LOCK = (acquire->release->LOCK).
U1 = (acquire -> use -> release -> U1).
U2 = (acquire -> use -> release -> U2).
|| SYSTEM = (u1:U1 || u2:U2 || {u1,u2}::LOCK).
```

Above allows for lock->use->release for either user but only one of them at a time.

Monitors and Semaphores

Monitor - A threadsafe class where each function is wrapped by a mutex. Essentially, only one process may access the class at a time. Entirely syntactic sugar. **Semaphore** - Essentially a mutex with a queue of processes

```
down(s): if s > 0 then
    decrement s
else
    block execution of calling process
up(s): if processes blocked on s then
    awaken one of them
else
    increment s

const Max = 3
range Int = 0..Max
SEMAPHORE(N=0) = SEMA[N],
SEMA[v:Int] = (up->SEMA[v+1]
    | when (v>0) down->SEMA[v-1]
    ).
LOOP = (mutex.down -> critical -> mutex.up -> LOOP
    <-> ).
|| SEMADEMO = (p[1..3]:LOOP || {p[1..3]}::mutex:
    <-> SEMAPHORE(1)).
```

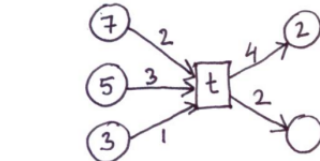
Bounded Buffer

A buffer with a fixed number of slots

```
BUFFER(N=5) = COUNT[0],
COUNT[i:0..N] = (when (i<N) put->COUNT[i+1]
    | when (i>0) get->COUNT[i-1]
    ).
PRODUCER = (put->PRODUCER).
CONSUMER = (get->CONSUMER).
|| BOUNDEDBUFFER = (PRODUCER || BUFFER(5) || CONSUMER).
```

Nested Monitor Problem P/T nets

Each place in a P/T net can hold multiple tokens. Each transition has a weight, w , associated with it. If it is an input transition, firing takes w tokens from the input place. If it is an output transition, firing adds w tokens to the output place. An action can only be fired if enough input tokens are present in all input



places.

Deadlocks

Dining Philosophers Problem

Simple minded construction:

```
FORK = (get -> put -> FORK).
PHIL = (think -> right.get -> left.get -> eat ->
    <-> right.put -> left.put -> PHIL).
|| DINERS(N = 5) = forall [i : 1..N] (phil[i] : PHIL
    <-> | {phil[i].right, phil[(i % 5) + 1].
    <-> left} :: FORK
```

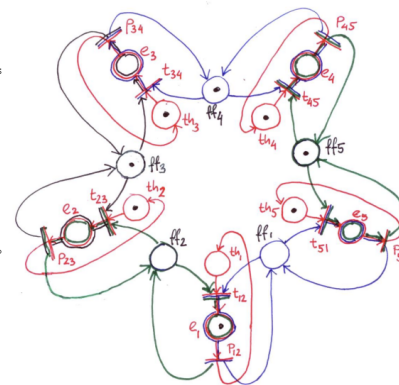
Solution 1 - Add asymmetry into the composition, where 1, 3, 5 always perform 'left.get -> right.get', while 2, 4 always perform 'right.get -> left.get'.

```
PHIL = (when (i=1|i=3|i=5) think -> left.get ->
    <-> right.get -> eat -> left.put -> right.
    <-> put -> PHIL
| when (i=2|i=4) think -> right.get -> left.
    <-> get -> eat -> right.put -> left.
    <-> put -> PHIL).
```

Solution 2 - Use a butler to prevent more than 4 philosophers from sitting at the table.

```
PHIL = (think -> sitdown -> right.get -> left.get
    <-> right.put -> left.put -> sitdown ->
    <-> getup -> PHIL).
BUTLER(K=4) = COUNT[0],
COUNT[i:1..4] = (when (i<K) sitdown -> COUNT[i+1]
    <-> getup -> COUNT[i-1]).
|| DINERS(N=5) ...
|| {phil[i:1..N]}::BUTLER(K=4)).
```

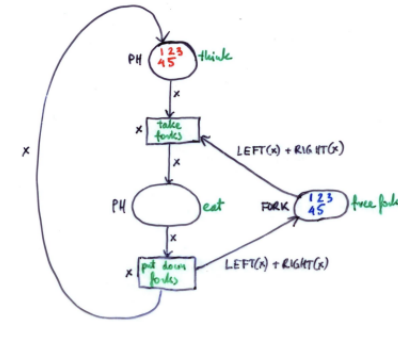
Solution 3 - Use Simultaneity



Only fire a transition if both forks are available

Coloured Petri Nets

'Colours' are simply types of tokens that are passed around the petri net. Paths to transitions are either labeled with variables or functions that transform one of the input variables into the object to remove from a state.



```
colour PH = with ph1 | ph2 | ph3 | ph4 | ph5
colour Fork = with f1 | f2 | f3 | f4 | f5
LEFT : PH -> FORK, RIGHT : PH -> FORK
var x : PH
fun LEFT x = case of ph1 => f2 | ph2 => f3 | ph3
    => f4 | ph4 => f5 | ph5 => f1
fun RIGHT x = case of ph1 => f1 | ph2 => f2 | ph3
    => f3 | ph4 => f4 | ph5 => f5
```

Semaphores and Extensions

Dijkstra's Semaphore Operations

C(s) - initial value of a semaphore variable s
ndown(s) - number of times down(s) was executed
nup(s) - number of times up(s) was executed
npdown(s) - number of times down(s) was passed
Then we define down and up:
down(s): ndown(s) = ndown(s)+1: if ndown(s) <= nup(s) + C(s) then npdown(s) = npdown(s) + 1;
up(s): if ndown(s) > nup(s) + C(s) then npdown(s) = npdown(s) + 1; nup(s) = nup(s)+1;

Theorem 1. $npdown(s) = \min(ndown(s), C(s) + nup(s))$

Multidimensional Semaphores of Agerwala

edown($s_1, \dots, s_n, s_{n+1}, \dots, s_{n+m}$): if for all $i, 1 \leq i \leq n, s_i > 0$ and for all $j, 1 \leq j \leq m, s_{n+j} = 0$ then for all $i, 1 \leq i \leq n, s_i = s_i - 1$ else block execution of calling processes
eup(s_1, s_2, \dots, s_n): if processes blocked on (s_1, \dots, s_n) then awaken all of them else for all $i, 1 \leq i \leq n, s_i = s_i + 1$

Inhibitor Nets

Add a circle to the transition side of an arc to make it an inhibitor arc. Now the transition can only be fired if the places connected by inhibitor arcs are empty.

Smokers' Problem

3 Smokers each have an unlimited type of either tobacco, cigarette paper, matches. 2 ingredients are placed on the table, the smoker with the third ingredient needed should pick up the ingredients, make a cigarette, and smoke it. Next set of ingredients won't be placed until smoking is completed.

Simple-minded Solution

```
SMOKER_T=( get_paper -> get_match->roll_cigarette
    <-> smoke_cigarette -> SMOKER_T ).
SMOKER_P=( get_tobacco -> get_match->
    <-> roll_cigarette -> smoke_cigarette ->
    <-> SMOKER_P ).
SMOKER_M=( get_tobacco -> get_paper->
    <-> roll_cigarette -> smoke_cigarette ->
    <-> SMOKER_M ).
TOBACCO = ( delivered -> picked -> TOBACCO ).
PAPER = ( delivered -> picked -> PAPER ).
MATCH = ( delivered -> picked -> MATCH ).
AGENT_T = (can_deliver -> deliver_paper ->
    <-> deliver_match -> AGENT_T ).
AGENT_P = (can_deliver -> deliver_match ->
    <-> deliver_tobacco -> AGENT_P ).
AGENT_M = (can_deliver -> deliver_tobacco ->
    <-> deliver_paper -> AGENT_M ).
RULE = (can_deliver -> smoking_completed -> RULE )
|| SMOKERS = (s_t:SMOKER_T || s_p:SMOKER_P || s_m:
    <-> SMOKER_M ).
```

```
||RESOURCES = ({s_m,s_p}::TOBACCO || {s_t,s_m}::
    <-> PAPER || {s_t,s_p}::MATCH ).
||AGENT_RULE = ({s_m,s_p,s_t}::RULE || {s_m,s_p}::
    <-> AGENT_T || {s_m,s_t}::AGENT_P ||
    {s_t,s_p}::AGENT_M ).
||CIG_SMOKERS = (SMOKERS || RESOURCES ||
    <-> AGENT_RULE)/
{s_t.get_paper/s_t.picked,
s_m.get_paper/s_m.picked,
s_p.get_paper/s_p.picked,
s_t.deliver_paper/s_t.delivered,
s_m.deliver_paper/s_m.delivered,
s_p.deliver_paper/s_p.delivered,
s_t.smoking_completed/s_t.smoke_cigarette,
s_m.smoking_completed/s_m.smoke_cigarette,
s_p.smoking_completed/s_p.smoke_cigarette}.
```

Property (safety)

property CORRECT_PICKUP =
(s_t.get_paper->s_t.get_match->CORRECT_PICKUP
| s_p.get_tobacco->s_p.get_match->CORRECT_PICKUP
| s_m.get_tobacco->s_m.get_paper->CORRECT_PICKUP).

Ask first, do later

```
SMOKER_T=( no_tobacco -> get_paper -> get_match->
    <-> roll_cigarette ->
    <-> smoke_cigarette -> SMOKER_T)
SMOKER_P=( no_paper -> get_tobacco -> get_match->
    <-> roll_cigarette ->
    <-> smoke_cigarette -> SMOKER_P)
SMOKER_M=( no_match -> get_tobacco -> get_paper->
    <-> roll_cigarette ->
    <-> smoke_cigarette -> SMOKER_T)
TOBACCO = ( delivered -> picked -> TOBACCO )
PAPER = ( delivered -> picked -> PAPER )
MATCH = ( delivered -> picked -> MATCH )
AGENT_T = (can_deliver -> no_tobacco ->
    <-> deliver_paper->deliver_match->AGENT_T)
AGENT_P = (can_deliver -> no_paper ->
    <-> deliver_match->deliver_tobacco->AGENT_P
    <-> )
AGENT_M = (can_deliver -> no_match ->
    <-> deliver_tobacco->deliver_paper->AGENT_M
    <-> nup(s))
RULE = (can_deliver -> smoking_completed -> RULE )
SMOKERS = s_t:SMOKER_T || s_p:SMOKER_P || s_m:
    <-> SMOKER_M
RESOURCES = {s_m,s_p}::TOBACCO || {s_t,s_m}::PAPER
    <-> || {s_t,s_p}::MATCH
AGENT_RULE = {s_m,s_p,s_t}::RULE || {s_m,s_p}::
    <-> AGENT_T || {s_m,s_t}::AGENT_P || {s_t,
    <-> s_p}::AGENT_M
CIG_SMOKERS = (SMOKERS || RESOURCES || AGENT_RULE)
    <-> /
{s_t.get_paper/s_t.picked,
s_m.get_paper/s_m.picked,
s_p.get_paper/s_p.picked,
s_t.deliver_paper/s_t.delivered,
s_m.deliver_paper/s_m.delivered,
s_p.deliver_paper/s_p.delivered,
s_t.smoking_completed/s_t.smoke_cigarette,
s_m.smoking_completed/s_m.smoke_cigarette,
s_p.smoking_completed/s_p.smoke_cigarette}.
```

Safety and Liveness

Safety - asserts that nothing bad happens

Safety Property P - defines a process that asserts any trace including the actions in the alphabet of P is accepted by P , otherwise they are transitions to the ERROR state, safety checks are compositional hence they should be composed with the appropriate (sub)system

Liveness - asserts that something good eventually happens

Progress Property - asserts that it is always the case that a particular action is eventually executed, opposite of starvation, progress checks are not compositional hence they should be conducted after safety checks

Starvation - situation in which an action is never executed

Terminal Set of States - set of states in which every state is reachable from every other state in the set and there is no transition from within to set to any state outside the set

Priority - specifies actions that have a higher/lower priority than any other action in the alphabet of some state

