Factor	Prefix	Symbol
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

Intro

System: Combination of interacting elements organized to achieve a stated purpose or meet an operational need. **System Boundary**: Defines separation of system of interest and operating environment. **Interface**: Input / Output that flows across system boundaries. **Mission**: A problem that a system intends to solve. Should be clearly defined by the stakeholder acquiring the system.

Life Cycle Conceptualization: Defining requirements. E.g. what does the market want? What type of product are we building? What features do we want? **Realization**: Taking requirements, making design. Building factories, tooling, prototypes, enter full production. **Utilization**: Processes during the use of the system. Marketing / sales, maintenance, warranty, recalls, feedback. **Retirement**: End-of-life services. Recycling, special disposal, updating / disposing of tooling.

Systems Engineering Overview

Top-down approach to design, development, operation of systems. Iterative, repeats for each lower level until individual elements are defined. Used because good planning and clear definition of deliverables makes sure that stakeholder needs and engineering solutions are aligned.

Project Life Cycle

Pre-Phase A: Concept Studies. Generate ideas for new missions, draft system concepts. Establish needs, goals, objectives. Assess feasibility. **Phase A**: Concept and Technology Development. Develop mission into requirements, system architecture. Identify technology development required. **Phase B**: Preliminary Design. Develop system concept into design solution meeting mission needs. Finish technology development. **Phase C**: Final Design and Fabrication. Complete detailed design, procure or fabricate components. Establish procedures, controls for manufacture. **Phase D**: Assembly, Integration, Test, Launch. Assemble subsystems, integrate, test to verify and validate system against requirements, deploy. **Phase E**: Operations and Sustainment. Put system into service. **Phase F**: Closeout. End of life operations, analysis. Retire system.

Stakeholders: Groups that are affected or have an interest/stake in program or project. **Need**: A single statement clearly describing what the customer wants. Domain of customer. Independent of specific solution / implementation. System is not the need, but response to need. E.g. "The Government of Canada needs a more effective means of monitoring shipping traffic in the arctic." **Goals**: Elaboration of need, setting specific expectations on system. Qualitative expressions of what system will accomplish. E.g. "Provide rapid detection of vessel traffic in Canadian arctic waters." **Objectives**: Specific target outputs system must achieve. Specific, Measurable, Attainable, Relevant, Time-Bound. Quantitative extension of goals. E.g. "Detect any surface vessel entering Canadian arctic territorial waters within 1 hour." **Measures of Effectiveness**: Metrics to judge whether a mission is successful in meeting objectives / achieving goals. Stated from stakeholder's POV. E.g. "No undetected vessels appearing inside 50 nm of Canadian arctic territorial waters." (Not directly observable by design team, each MOE assigned 1+ *Measures of Performance*, defining level of perf system must meet to enable MOE.) **Constraints**: Boundaries placed on system design. Usually one of technical, performance, resources, environmental, schedule, cost, regulatory, organizational.

Concept of Operations: Document that outlines high-level vision & strategy for use & operation of a system to achieve intended goals. Who are the stakeholders? What is the mission? What is the proposed solution? How will it be used? Where will it be used? By whom? Over what time? *Context Diagram*: Shows mission, system scope. *Timeline*: Shows time sequence of operations. Can identify weak spots, e.g. unacceptable gaps in coverage. **Mission Requirements**: Formal statements defining capabilities, functions, performance, operating condition for system to meet goals, objectives. Requirements can be validated (demonstrated to be true). Mission requirements starting point for system requirements.

Requirements

Requirements Engineering

Process of translating stakeholder expectations into technical statements usable by design team to make a solution that meets the original need. Bridges gap between stakeholder expectations, design teams technical instructions.

Why requirements

Tell us what our system needs to do. Tell us scope of system. Give opportunity for every stakeholders' input to be captured. Help structure, scope work to be done, estimating effort and cost. Allow us to track project progress. How we know we are done, how well we did.

Separate from the "Needs View", directly define the input to design.

Requirement = input to design.
Specification = output of design.

Internal Requirements

Self-imposed, usually during R&D, early product development. Will evolve, change during early project life-cycle.

External Requirements

Defined by the customer, contractor must abide by these. Changes are more difficult and require more evidence, approvals. Various formats, stated order of precedence. E.g. contract, statement of work, technical requirements, product assurance requirements.

Requirement

Formally written, agreed upon statement of what the system must do, a quality it must posess, or a constraint it must operate under in order to meet the need. "The <system name> shall <system response>"

Shall: a statement of requirement. This must be met. **Should:** a statement of a goal or non-mandatory request. Desired, but not required. **Will:** a statement of fact, declaration of purpose.

Functional Requirements:

What functions need to be performed to accomplish objective. System as a "black box" of functions. Focus on *what* needs to be done, not *how* to do it.

Non-functional Requirements:

Other properties the system must posess, constraints it must operate under. Performance, environment, interfaces, constraints, "-ilities", training, personnel, safety.

Performance: How well the system should perform functions. How much / little, how far, how fast, how many, how often... E.g. bit error rates.

Environmental Requirements: Requirements defining operating system of system. E.g. shock exposure levels, operating temperatures.

Interface Requirements: How elements interface with other elements internal / external to system. (Accepting inputs, providing outputs are functional, but specifics on how the interface should work are not) E.g. what standard of communication is used between systems.

Constraints Limitations, boundaries, conditions imposed on system by stakeholders. Include cost, timeline, physical dimension, quantity (size, weight, power), rules and regulations.

-ilities Deal with life-cycle considerations product quality, other stakeholders. Relability, availability, scalability, maintainability, operabiliy, supportability, security, manufacturability, interoperability, etc. Major design, cost drivers.

Good Requirements

Good requirements should be

- Needed (should be necessary, sufficient to specify system. Nothing extraneous.)
- Verifiable (must have a clear pass/fail criteria, a method to determine if it is met)
- Attainable (no point in requiring the impossible)
- Traceable (must be linked from the lowest component to the highest need)

Good requirements are also good comms: Single thought, concise, simple ad consistent, grammatically correct.

Requirement Documentation

Requirement ID, Title, Requirement Text, Rationale, Verification, Tracability, Notes.

Requirement Rationale

Additional information on intent, context for a requirement, helps with interpretation. Reason for the requirement. Documents assumptions made when writing requirement. Link to supporting information outside requirement set, e.g. architecture decisions, ConOps, trade studies, customer discussions, etc...

Requirement Verification

Reqs need to be verified.

- Inspection: visual examination of product, supporting documentation
- Demonstration: using product to demonstrate requirement is met
- Analysis: use of modelling, simulation, analytical techniques to predict a product will meet requirement
- Test: highly controlled use, measurement of product to compare to pass/fail criteria

Requirement Traceability

Ability to trace every requirement to it's source. Most come from other requirements, not always. ALso applies to ConOps, architecture documents, trade studies, analysis, other documentation.

Forwards Traceability: Relationship of parent requirements down to children. **Backwards Traceability:** Relationship of child requirements up to their parents.

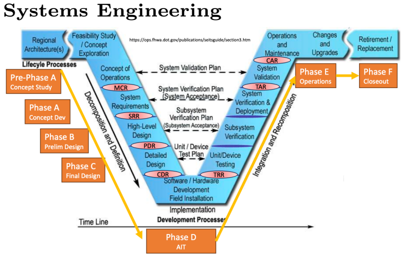
Requirement Validation

Ensure the requirements capture stakeholder's expectation. Ensure are complete and correct, accurately capturing the stakeholder needs, necessary, achievable, not duplicated or over-specified, verifiable.

Why manage requirements

Requirements will evolve as system evolves. Complex projects have hundreds of requirements, allocated across many elements, multiple developers.

Systems Engineering



System Design Process: Analysis, Synthesis, Evaluation.

ConOps: Operations Concept (aka a diagram)

NASA System Design Process 1. Stakeholder Expectation Definition: Identify stakeholders, expectations, requirements. Capture expectations in ConOps. Identify Measures of Effectiveness. **2. Technical Requirements Definition:** Turn stakeholder expectations into formal technical statements used to further decompose, define system. Identify system scope (what are we responsible for, what are we not responsible for), system boundary (physical manifestation of scope). Identify interfaces. Define functions to use system, as identified in ConOps. **Elicitation:** Drawing out, e.g. stakeholder telling us, recieving it from a source. **Elaboration:** Further develop existing requirements using Decomposition (breaking it down into smaller parts) or Derivation (making inferences and creating new requirements). **3. Logical Decomposition:** Problem has been defined, now prepare to solve it. Decompose problem into smaller parts (logical or functional). Proposing architecture as candidate solution to decomposed problem. Allocating functional decomposition to elements in architecture. Deriving further requirements. Evaluating proposed solution. Iterating to find best overall solution for need. Finalizing all requirements to bring to design. **4. Design Solution Definition:** Translate requirements into design. (Most course work / lab work, here is a problem solv it.) Propose and analyze multiple potential solutions. Selected alternative fully defined into complete design solution.

Launch Environment

Thrust Equation: $T = \dot{m}_e v_e + (p_e - p_a)A_e$
Rocketry Equation: $\Delta v = I_{sp} g_0 \ln \frac{m_0}{m_f}$ **Design for Launch** Max Q: peak dynamic pressure. Rocket vibration from thrust, static load/random vibration, structural resonance. Acoustic vibration from atmospheric forces. Resonance and dampening: Q is ratio of output displacement to input. Generally aluminum is used. Thermal baance, we want $Q_{in} = Q_{out}$, where $Q_{out} = Q_{solar} + Q_{albedo} + Q_{EarthIR} + Q_{spacecraft}$. (Albedo is reflection of sun from Earth.)

Test thermal properties using TVAC (thermal vaccum chamber), heat lamps to simulate solar / albedo loading.

Quality

Quality: How to make sure that your solution meets safety/reliability standards? Guard against failure. Plan -> Do -> Check -> Act -> Repeat. Performed by Product Assurance -> Quality Assurance -> Quality Control -> Audit. QC: Inspecting work to ensure quality is maintained. QA: Process control, ensuring process is in place, maintain documentation and traceability for audits. Manufacturing, test data to verify something conforms to engineering data. Process to identify, prevent defects. Plans, implements systematic activities to provide confidence that all quality requirements will be fulfilled. PA: Make sure product meets specifications, including quality, assurance. "Voice of customer". Ensure project is built per process using "stuff" that meets customer requirements. Proposal -> design -> manu -> test -> deliv -> commissioning. Certificat of Conformance (indicates deliverable built and tested per reqs.)

PA Requirement e.g.: EEE Components (Electrical, Electronic, or Electromechanical) must either be on an approved list or be approved, must be purchased from approved manufacturers, components must also pass specifications, must pass qualification.

E.g. NASA EEE-INST-002: Level 1: Lowest risk, highest reliability. Level 2: Low to moderate risk. Level 3: Higher risk. Commercial. Higher level = higher standards, testing, can be an order of magnitude more expensive. Lower quality components need to undergo additional testing to be accepted.

5. Product Realization Design is done, SE switches to "bottom-up", building smallest components first up to largest. Usually either build, buy, or reuse, based on skills, capability, experience, cost, etc. Can buy by paying a contractor to build (more requirements need to be exchanged), or purchase COTS (existing stuff that is higher risk). COTS is hard to qualify, lower performance. Can test new parts on case-by-case basis. Reusing is usually attractive, especially if it has worked in space before.

Assembly: joining individual components together to make higher level products.
Integration: combining elements to achieve a higher level function. Must include some level of "using" product to enable the purpose. Integration ensures different subsystems work together properly. **Test:** Test as you go, make sure each part works.

6. Product Integration Assemble, Integrate, Test until full system is done. Systems plans for AIT in design, sees it through here. Quality makes sure build quality is maintainted, tests are performed properly.

7. Product Verification Did we build it right?

Model philosiphies Breadboard: test model, cheap components, partial designs. Engineering model: CLose to flight, cheaper parts, less functionality. Engineering Qualification Model: Close to flight, cheaper parts, full functionality. Qualification Model: Identical to flight, flight component, flight process. Proto-Flight Model: Full functionality, qualified parts, materials, process. Tested to higher requirements to check margin. Only on first flight deliverable. Flight model: Same as proto-flight model, tested to normal requirements.

8. Product Validation Did we build the right system? Does it meet the needs of the customers / users?

Validation vs Verification Verification: Did we build the system how we said we would? Validation: Stakeholder expectations (MOEs, and MOPs)

9. Product Transition

10-17. Technical Management

Satellite Systems

Primary Uses of Space: Communication, navigation, observation, exploration, and experimentation. Examples include communication systems like Intelsat 40e and SpaceX Starlink; navigation systems like GPS; observation systems such as Radarsat 2 and Sentinel; and exploration missions like Artemis/Orion, ISS, and the James Webb Space Telescope (JWST). **A Systems View of Satellites:** Satellites extend the system boundary to space, requiring enabling products such as launch vehicles and ground stations. They host payloads to perform system functions in orbit. **System Design Drivers:** Key drivers include: Cost, which impacts parts, testing equipment, and personnel; Schedule, limiting design, testing, and component availability; Orbit, affecting thermal, radiation, and atmospheric conditions; Lifetime, driving redundancy and reliability requirements; Payload, supporting interfaces, power, orientation, and data flow; Volume and Mass, dictated by the launcher; and Power, determined by orbit and influencing solar panels, batteries, and power distribution. **Satellite Functional Hierarchy:** Satellites are divided into two primary elements: The **Payload**, which provides the mission capability (e.g., GPS clocks, imaging sensors); and The **Bus**, which supports the payload with power, thermal control, orbit and attitude control, and communication. **Key Subsystems:** These include: **Structure**, which supports satellite components and interfaces with the launcher; **Thermal Control**, which maintains system temperatures using radiators and heat pipes; **Power**, which provides energy via solar panels, batteries, and distribution systems; **Propulsion**, which adjusts orbit and orientation using chemical, cold gas, or electric systems; **Attitude Control**, which ensures stability and pointing with sensors and actuators (e.g., reaction wheels, star trackers); **Data Handling**, which manages commands, telemetry, and mission data; and **Communications**, which links the satellite to ground stations for telemetry and payload data.

Space Environment

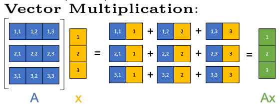
1. Near-Earth Radiation Environment: Van Allen Belts: Inner Belt (high-energy protons/electrons, ~1,000-6,000 km). Outer Belt (high-energy electrons, ~9,000 km to beyond geostationary orbit). **South Atlantic Anomaly (SAA):** Region of lower magnetic field where radiation penetrates to lower altitudes; can disrupt electronics. **Solar Influence:** Solar cycles (~11 years): Maximum increases flares and CMEs; minimum increases trapped particles. **2. Cosmic Rays:** High-energy particles from outside the solar system, partially deflected by Earth's magnetic field. **3. Radiation Effects:**

On Materials: Degrades optics, structural components, and electrical parts. **Types of Effects:** TID (long-term degradation), SEE (immediate damage, e.g., bit flips, hardware failures). **4. Mitigation Strategies: Radiation Shielding:** Straight-line and Monte Carlo models (e.g., Geant). **Designing for Tolerance:** Use radiation-hardened components and fail-safe designs. **5. Thermal Considerations: LEO:** Rapid cycling between sunlight ($\sim +120^\circ$C) and shadow ($\sim -100^\circ$C) every 90 minutes. **GEO:** Longer thermal transition periods due to 24-hour orbits. **6. Plasma and Charging:** Plasma from solar storms or Earth's magnetosphere can cause charging and electrostatic discharge, damaging systems.

Math

Dot Product: $\vec{a}\cdot\vec{b} = a_x b_x + a_y b_y + a_z b_z = |\vec{a}||\vec{b}|\cos\theta$; $|\vec{a}|^2 = \vec{a}\cdot\vec{a}$. **Projection**: $proj_{\vec{b}}\vec{a} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}\frac{\vec{b}}{|\vec{b}|} = \frac{\vec{a}\cdot\vec{b}}{\vec{b}\cdot\vec{b}}\hat{b}$; $\vec{a}_{\perp\vec{b}} = \vec{a} - proj_{\vec{b}}\vec{a}$.

Cross Product: $\vec{a}\times\vec{b} = (a_y b_z - a_z b_y)\hat{i} - (a_x b_z - a_z b_x)\hat{j} + (a_x b_y - a_y b_x)\hat{k}$; $\hat{n}_{ab} = \frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$; $\vec{a}\times\vec{b} = |a||b|\sin\theta\,\hat{n}_{ab}$. **Matrix x Vector Multiplication**:



Orbital Mechanics

2D

Kepler's Laws K1 - Orbit of planet is ellipse, Sun at one of two foci. K2 - Line segment joining planet, Sun sweeps out equal areas during equal intervals of time. K3 - Square of planet's orbital period proportional to cube of length of semi-major axis of orbit.

Forces

$\vec{F} = m\vec{a} = \frac{d\vec{p}}{dt}$. $\vec{p} = m\vec{v}$. **Impulse on mass** m: (change in momentum when acted upon by a force) $I = \int_{t_1}^{t_2}\vec{F}\,dt = m\vec{v}_2 - m\vec{v}_1$. $\Delta\vec{v} = I/m = F_{net}\Delta t/m$ (if F_{net} const).

Gravitational Constant: $G = 6.67430\times10^{-11}\ \frac{\text{Nm}^2}{\text{kg}^2}$.

Rotation, Torque, Angular Momentum

Torque (moment) on mass m: $\vec{M_0} = \vec{r}\times\vec{F} = \frac{d\vec{H_0}}{dt}$ **Angular Momentum:** $\vec{H_0} = \vec{r}\times m\vec{v} = \vec{r}\times\vec{p}$

Orbits

Two-body equation of motion: $\mu = G(m_1+m_2)$ (gravitational param); $\ddot{\vec{r}} = -\frac{G(m_1+m_2)}{r^3}\vec{r} = -\frac{\mu}{r^3}\vec{r}$.

Angular Momentum: $\vec{h} = \vec{r}\times\vec{v}$ kg m s^{-1} (convert to kg km s^{-1} if needed)

Eccentricity vector: $\vec{e} = \frac{\vec{C}}{\mu}$ **Orbit Equation:**

$$r = \frac{h^2}{mu}\frac{1}{1+e\cos\theta}$$

Circular Orbit $e = 0$

$r = h^2/\mu = \mu/v^2$; $h = rv_\perp$; $v = v_\perp$; $v_{circ} = \sqrt{\frac{\mu}{r}}$; $T_{circ} = \frac{2\pi}{\sqrt{mu}r^{3/2}}$

Orbital Constants

$G = 6.67430\times10^{-20}$ N km^2 kg^{-2}; $R_E = 6378$ km; $M_E = 5.97219\times10^{24}$ kg; $\mu = G(M_e+M_s)\approx G(M_e)\approx 398600$ km^3 s^{-2};

Elliptical Orbit $0 < e < 1$

$r = \frac{h^2}{\mu}\frac{1}{1+e\cos\theta} = \frac{a(1-e^2)}{1+e\cos\theta}$.

Periapsis: Closest approach.

Apoapsis: Furthest approach. $a+c$ away from the mass at focal point.

Semimajor axis: $a = \frac{h^2}{\mu}\frac{1}{1-e^2}$ (the longer one)

Semiminor axis: $b = a\sqrt{1-e^2}$ (the shorter one)

Linear eccentricity: $c = ae$

Semilatus rectum: $p = \frac{h^2}{\mu}$

True anomaly: θ, the degree of rotation from periapsis.

Specific Energy of $m2$ **(constant)**: $\epsilon = -\frac{1}{2}\frac{\mu}{h^2}(1-e^2)$.

Orbit Period: $T_{ellipse} = \frac{2\pi ab}{h} = \frac{2\pi a^{3/2}}{\sqrt{\mu}}$, where $b = a\sqrt{1-e^2}$, $h = \sqrt{\mu a(1-e^2)}$

Parabolic Trajectory $e = 1$, *Hyperbolic Trajectory* $e > 1$

$v_{esc} = \sqrt{2}v_{circ}$, arrive at inifinity with 0 velocity. $v_\infty^2 = v^2 - v_{esc}^2$, arrive at inifinity with velocity v_∞.

Perifocal Frame

Coordinate frame where orbit lies in XY plane.

- Centered at the focus
- X axis points along apse line to periapsis (0 true anomaly)
- Y axis along semilatus rectum
- Z axis in normal direction of angular momentum $\vec{\mathbf{h}}$

$r = \frac{h^2}{\mu}\frac{1}{1+e\cos(\theta)}$, $x = r\cos\theta$, $y = r\sin\theta$

$\vec{\mathbf{r}} = x\,\hat{\mathbf{p}} + y\,\hat{\mathbf{q}}$

$\vec{\mathbf{r}} = \frac{h^2}{\mu}\frac{1}{1+e\cos(\theta)}(\cos(\theta)\,\hat{\mathbf{p}} + \sin(\theta)\,\hat{\mathbf{q}})$

$\vec{\mathbf{v}} = \dot{x}\,\hat{\mathbf{p}} + \dot{y}\,\hat{\mathbf{q}}$

$\vec{\mathbf{v}} = \frac{\mu}{h}(-\sin(\theta)\,\hat{\mathbf{p}} + (e+\cos(\theta))\,\hat{\mathbf{q}})$

Orbit Relationships

$2a = r_a + r_p$; $e = \frac{r_a - r_p}{r_a + r_p}$, $\frac{r_p}{r_a} = \frac{1-e}{1+e}$;

$a = \frac{h^2}{\mu}\frac{1}{1-e^2}$;

$h = \sqrt{\mu a(1-e^2)} = r_a v_{\perp a} = r_p v_{\perp p}$;

$r_a = \frac{h^2}{\mu}\frac{1}{1-e}$; $r_p = \frac{h^2}{\mu}\frac{1}{1+e}$;

$v_\perp = \frac{\mu}{h}(1+e\cos\theta)$ $v_r = \frac{\mu}{h}e\sin\theta$

Orbit Energy

Lagrange Coefficients

$h = |\vec{r_0}\times\vec{v_0}|$; $v_{r0} = \vec{v_0}\cdot\frac{\vec{r_0}}{r_0}$;

$r = \frac{h^2}{\mu}\frac{1}{1+\left(\frac{h^2}{\mu r_0}-1\right)\cos(\Delta\theta) - h\frac{v_{r0}}{\mu}\sin(\Delta\theta)}$

$f = 1 - \frac{\mu r}{h^2}(1-\cos(\Delta\theta))$, $g = \frac{rr_0}{h}\sin(\Delta\theta)$

$\dot{f} = \frac{\mu}{h}\frac{1-\cos(\Delta\theta)}{\sin(\Delta\theta)}\left[\frac{\mu}{h^2}(1-\cos(\Delta\theta)) - \frac{1}{r_0} - \frac{1}{r}\right]$

$\dot{g} = 1 - \frac{\mu r_0}{h^2}(1-\cos(\Delta\theta))$

Therefore, $\vec{r} = f\vec{r_0} + g\vec{v_0}$; $\vec{v} = \dot{f}\vec{r_0} + \dot{g}\vec{v_0}$

Solving for eccentricity, initial true anomaly: $r_0 = \frac{h^2}{\mu}\frac{1}{1+e\cos(\theta_0)}$ $v_{r0} = \frac{\mu}{h}e\sin(\theta_0)$

Orbit Position as a Function of Time

Circle: $\theta(t) = \frac{\mu^2}{h^3}t$, $t(\theta) = \frac{h^3}{\mu^2}\theta$

Elliptical Orbit: $M_e = t\frac{\mu^2}{h^3}(1-e^2)^{3/2}$

$E = 2\tan^{-1}\left(\sqrt{\frac{1-e}{1+3}}\tan\left(\frac{\theta}{2}\right)\right)$

$e\sin E = \frac{e\sqrt{1-e^2}\sin(\theta)}{1+e\cos(\theta)}$

$M_e = E - e\sin E = \frac{2\pi t}{T_{orbit}}$

(given θ solve for t directly, or $\theta\to E\to M_e\to t$)

3D

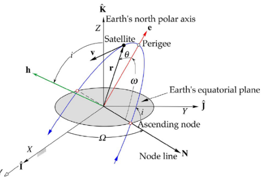
Sidereal Rate: Rotation rate of Earth relative to "fixed stars", $\omega_E = 7.292115\times10^{-5}$ rad/s.

Right Ascention: Degrees east along equator from vernal equinox. **Declination:** Latitude north from equator.

Geocentric Equatorial Frame

Coordinate frame is fixed, used to rep satellite position, velocity. **X axis** along direction of vernal equinox (at chosen epoch!) **Z axis** along Earth's axis of rotation (at chosen epoch!) **Y axis** completes cartesian coordinate system. Precession, Nutation of the Earth spin axis causes it to drift over time, so epoch matters. (Precession: Earth's axis rotating around a central axis, Nutation: variations / wiggles in precession)

Orbital Parameters



Given Position and Velocity vectors, we can calculate orbital elements:

1. Distance: $r = \sqrt{\vec{r}\cdot\vec{r}}$
2. Speed: $v = \sqrt{\vec{v}\cdot\vec{v}}$
3. Radial velocity: $v_r = \frac{\vec{r}}{r}\cdot\vec{v}$ ($v_r > 0$ away from perigee, $v_r < 0$ towards perigee)
4. Specific angular momentum: $\vec{h} = \vec{r}\times\vec{v}$ (magnitude $h = \sqrt{\vec{h}\cdot\vec{h}}$)
5. Inclination: $i = \cos^{-1}(h_Z/h)$ ($0^\circ \le i < 90^\circ$: prograde orbit (same direction as Earth rotation), $90^\circ < i \le 180^\circ$: retrograde orbit (opposite direction to Earth rotation))
6. Node line: $\vec{N} = \hat{K}\times\vec{h}$ (Magnitude $N = \sqrt{\vec{N}\cdot\vec{N}}$)
7. RAAN: $\Omega = \cos^{-1}(N_X/N)$ ($N_Y \ge 0, \Omega = \cos^{-1}(N_X/N)$, $N_Y < 0, \Omega = 360^\circ - \cos^{-1}(N_X/N)$)
8. Eccentricity vector: $\vec{e} = \frac{\vec{v}\times\vec{h}}{\mu} - \frac{\vec{r}}{r} = \frac{1}{\mu}\left[\left(v^2 - \frac{\mu}{r}\right)\vec{r} - rv_r\vec{v}\right]$ (Magnitude: $e = \sqrt{\vec{e}\cdot\vec{e}}$)
9. Argument of perigee: $$\omega = \begin{cases} e_Z \ge 0 & \cos^{-1}\left(\frac{\vec{N}}{N}\cdot\frac{\vec{e}}{e}\right) \\ e_Z < 0 & 360^\circ - \cos^{-1}\left(\frac{\vec{N}}{N}\cdot\frac{\vec{e}}{e}\right) \end{cases}$$
10. True Anomaly: $$\theta = \begin{cases} v_r \ge 0 & \cos^{-1}\left(\frac{\vec{e}}{e}\cdot\frac{\vec{r}}{r}\right) \\ & \text{or } \cos^{-1}\left(\frac{1}{e}\left[\frac{h^2}{\mu r} - 1\right]\right) \\ v_r < 0 & 360^\circ - \cos^{-1}\left(\frac{\vec{e}}{e}\cdot\frac{\vec{r}}{r}\right) \\ & \text{or } 360^\circ - \cos^{-1}\left(\frac{1}{e}\left[\frac{h^2}{\mu r} - 1\right]\right) \end{cases}$$

Perifocal to ECI Frame

$$\vec{\mathbf{r}} = \frac{h^2}{\mu}\frac{1}{1+e\cos(\theta)}\begin{bmatrix}\cos(\theta)\\ \sin(\theta)\\ 0\end{bmatrix}$$

$$\vec{\mathbf{v}} = \frac{\mu}{h}\begin{bmatrix}-\sin(\theta)\\ e+\cos(\theta)\\ 0\end{bmatrix}$$

2D Rotations

3D Rotations

Euler Z-X-Z: $(\alpha,\beta,\gamma)\to R_Z(\gamma)R_X\beta R_Z(\alpha)$ | $\alpha\in[0^\circ,360^\circ]$, $\beta\in[0,180^\circ]$, $\gamma\in[0^\circ,360^\circ]$

$$\vec{v}_{x'y'z'} = \begin{bmatrix}\cos\gamma & \sin\gamma & 0\\ -\sin\gamma & \cos\gamma & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 0 & \cos\beta & \sin\beta\\ 0 & -\sin\beta & \cos\beta\end{bmatrix}\begin{bmatrix}\cos\alpha & \sin\alpha & 0\\ -\sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}\vec{v}_{xyz}$$

$$Q_{ZXZ} = \begin{bmatrix}-\sin\alpha\cos\beta\sin\gamma + \cos\alpha\cos\gamma & \cos\alpha\cos\beta\sin\gamma + \sin\alpha\cos\gamma & \sin\beta\sin\gamma\\ -\cos\alpha\cos\beta\sin\gamma - \cos\alpha\sin\gamma & \cos\alpha\cos\beta\cos\gamma - \sin\alpha\sin\gamma & \sin\beta\cos\gamma\\ \sin\alpha\sin\beta & -\cos\alpha\sin\beta & \cos\beta\end{bmatrix}$$

Yaw-Pitch-Roll: $(\alpha,\beta,\gamma)\to R_X(\gamma)R_Y(\beta)R_Z(\alpha)$ | $\alpha\in[0,360^\circ]$, $\beta\in(-90^\circ,90^\circ)$, $\gamma\in[0^\circ,360^\circ]$

$$\vec{v}_{x'y'z'} = \begin{bmatrix}1 & 0 & 0\\ 0 & \cos\gamma & \sin\gamma\\ 0 & -\sin\gamma & \cos\gamma\end{bmatrix}\begin{bmatrix}\cos\beta & 0 & -\sin\beta\\ 0 & 1 & 0\\ \sin\beta & 0 & \cos\beta\end{bmatrix}\begin{bmatrix}\cos\alpha & \sin\alpha & 0\\ -\sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}\vec{v}_{xyz}$$

$$Q_{YPR} = \begin{bmatrix}\cos\alpha\cos\beta & \sin\alpha\cos\beta & -\sin\beta\\ \cos\alpha\sin\beta\sin\gamma - \sin\alpha\cos\gamma & \sin\alpha\sin\beta\sin\gamma + \cos\alpha\cos\gamma & \cos\beta\sin\gamma\\ \cos\alpha\sin\beta\cos\gamma + \sin\alpha\sin\gamma & \sin\alpha\sin\beta\cos\gamma - \cos\alpha\sin\gamma & \cos\beta\cos\gamma\end{bmatrix}$$

Perifocal to Geocentric Equatorial

$Q_{GCE\to pfc} = R_Z(\omega)R_X(i)R_Z(\Omega)$

$$Q_{pfc\to GCE} = Q^T_{XYZ\to pqw} = R_Z(-\Omega)R_X(-i)R_Z(-\omega) = \begin{bmatrix}-\sin\omega\cos i\sin\Omega + \cos\omega\cos\Omega & -\cos\omega\cos i\sin\Omega - \sin\omega\cos\Omega & \sin i\sin\Omega\\ \cos\omega\cos i\sin\Omega + \cos\omega\sin\Omega & \cos\omega\cos i\cos\Omega - \sin\omega\sin\Omega & -\sin i\cos\Omega\\ \sin\omega\sin i & \cos\omega\sin i & \cos i\end{bmatrix}$$

Oblateness

Regression of Node:

$$\dot{\Omega} = -\left[\frac{3}{2}\frac{\sqrt{\mu}J_2R^2}{(1-e^2)^2a^{7/2}}\right]\cos i$$

Advance of Perigee:

$$\dot{\omega} = -\left[\frac{3}{2}\frac{\sqrt{\mu}J_2R^2}{(1-e^2)^2a^{7/2}}\right]\left(\frac{5}{2}\sin^2 i - 2\right)$$

Date-related 3D

Latitude and Longitude to Cartesian Coordinates (Radius of line from point at perpendicular height H above reference surface to polar axis)

$R_\phi = \frac{R_E}{\sqrt{1-(2f-f^2)\sin^2\phi}}$

$R_{xyz} = \begin{bmatrix}(R_\phi + H)\cos\phi\cos\Lambda\\ (R_\phi + H)\cos\phi\sin\Lambda\\ ((1-f^2)R_\phi + H)\sin\phi\end{bmatrix}$ H: Altitude of coordinate

$f = (R_E - R_P)/R_E$: Flattening factor of oblate spheroid

R_E: Equatorial radius of the Earth

R_P: Polar radius

ϕ: Geodic Latitude

Λ: Geodic Longitude

WGS84 Reference Ellipsoid

$R_E = 6\,378\,137.0$ m

$f = 1/298.257\,223\,563$

$R_P \approx 6\,356\,752.314\,245$ m

Julian Date Formula

$JD = JD_0 + \frac{UT}{24}$

$JD_0 = 367y - \left\lfloor\frac{7\left(y - \left\lfloor\frac{m+9}{12}\right\rfloor\right)}{4}\right\rfloor + \left\lfloor\frac{275m}{9}\right\rfloor + d + 1\,721\,013.5$

Sidereal Time

Centuries since J2000: $T_0 = \frac{JD_0 - 2\,451\,545}{36\,525}$

Greenwich sidereal time at 0 UT: $\theta_{G_0} = 100.460\,618\,4 + 36\,000.770\,04T_0 + 0.000\,387\,933T_0^2 - 2.583\times10^{-8}T_0^3(deg)$

Greenwich sidreal time at current time in UT: $\theta_G = \theta_{G_0} + 360.985\,647\,24\frac{UT}{24}$

Local sidereal time: $\theta_{SD} = \theta_G + \Lambda$.

To go from Earth-Centered-Earth_fixed to Earth-Centered-Inertial, we need to perform a Z axis frame rotation from ECEF to ECI by sidereal time. AKA, we need to rotate by $-\theta_G$.

Satellite Communications

Channel Capacity: $C = BW\log_2\left(1 + \frac{S}{N}\right)$ bit s^{-1} BW: bandwidth, S: signal, N: noise. SNR: $\frac{S}{N} = \frac{P_{sig}}{P_{noise}} = \frac{P_{sig}}{N_0 BW}$ (N_0 is power spectral density of noise (e.g. W Hz^{-1}). Usually associated with thermal noise in system,

Symbol Rate: $R_S = \frac{1}{T_S} = BW$ (inverse of time the symbol takes up)

Average Energy per Symbol: $E_S = \frac{1}{M}\sum_{i=1}^{M}\int_{-\infty}^{\infty}|s_i(t)|^2 dt$

Average Power: $P_s = E_S R_S$

Probability of Error:

$p_e = Q\left(\frac{s_1 - s_0}{\sigma_1 + \sigma_0}\right) = Q\left(\frac{d}{2\sigma}\right)$

$p_{e,OOK} = Q\left(\sqrt{\frac{E_b}{N_0}}\right) = Q(\sqrt{SNR_{lin}})$

$p_{e,BPSK} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = Q(\sqrt{2SNR_{lin}})$

Link Budget Equation

Base SI units: $P_{RX} = P_{TX}L_{TX}G_{TX}L_{FS}L_PG_{RX}L_{RX}$

Logarithmic units: $P_{RX}(dBm) = P_{TX}(dBm) - L_{TX}(dB) + G_{TX}(dB) - L_{FS}(dB) - L_P(dB) + G_{RX}(dB) - L_{RX}(dB)$

Linear to Logarithmic: $X_dB = 10\log_{10}X_{lin}$, and $P_{dBm} = 10\log_{10}\frac{P_{lin}}{1mW}$ (1 mW = 0.001 W)

Antenna Gain

RF antennas: $I(r) = \frac{P}{4\pi r^2}\frac{W}{m^2}$ Gain for all power concentrated in area S: $G = \frac{4\pi r^2}{S}$ E.g. Parabolic antenna (high directivity):

$G_{par} = \frac{4\pi A}{\lambda^2}e_A = \left(\frac{\pi d}{\lambda}\right)^2 e_A$ aperture diameter d, wavelength λ, aperture efficiency factor e_A.

Optical antenna directional gain scales with wavelength: $G_{Tx} = \left(\frac{\pi D}{\lambda}\right)^2$

Antenna Coverage

(not used in calc?) $\theta = \frac{kd}{\lambda}$, $G_{par} = \left(\frac{\pi k}{\theta}\right)^2 e_A$

Free Space Loss

$L_{free} = \left(\frac{\lambda}{4\pi d}\right)^2$, $L_{free_{DB}} = 20\log_{10}\left(\frac{\lambda}{4\pi d}\right)$.

Pointing Loss

RF Antennas

1/2 power BW: $\theta_{3dB}\approx\frac{70^\circ\lambda}{D}$, and pointing loss: $L_{point}(dB)\approx -12\left(\frac{\Delta\theta}{\theta_{3dB}}\right)^2$

Optical Antennas

$1/e^2$ intensity profile BW: $\theta_{div}\approx\frac{\lambda}{\pi\omega_0}$, and pointing loss: $L_{point}(dB)\approx -8.7\left(\frac{\Delta\theta}{\theta_{div}}\right)^2$

Reciever Gain Same as above for transmitter, just update variables for reciever antenna.

Reciever Noise

$P_{th,RF} = k_bTBW$, or in dBm, $P_{th}(dbm) = -174\frac{dB}{Hz} + 10\log_1 0BW$

Noise Figure

N_f, How much extra noise is injected by reciever electronics.

Desired SNR

$SNR_{BER} = P_{Rx} - P_{noise} = P_{Rx} + 174 - 10\log_{10}BW - N_f$

Because optical interferometers are hard, $SNR_{opt} = \bar{n_s} = \frac{P_{Rx}}{h\nu R_s}$ $h = 6.626\,070\,15\times10^{-34}$ J Hz^{-1}

Satellite Attitude & Orbit Control Systems

ADCS Subsystem: System to estimate spacecraft body orientation, determine error, process spacecraft dynamics to formulate corrective commands, command actuators to perform adjustements.

Attitude Sensing

Estimate of current spacecraft orientation from sensor measurements

Sun Sensor

Orients at the sun. $\alpha = \tan^{-1}\frac{\Delta X}{h}$, $\beta = \tan^{-1}\frac{\Delta Y}{h}$.

Horizon Sensor

Uses local horizon to orient towards Earth Earth is warm versus space. Horizon depends on altitude, so use a wider FOV for lower orbits.

Star tracker

$star_{pos}(x_i, y_i) = (f\tan\theta, f\tan\phi)$.

Spacecraft Dynamics

How to go from orbital parameters to state vectors?

1. Given orbital parameters, calculate r, v in perifocal frame: $$\{\mathbf{r}\}_{\bar{x}} = \frac{h^2}{\mu}\frac{1}{1+e\cos\theta}\begin{Bmatrix}\cos\theta\\ \sin\theta\\ 0\end{Bmatrix} \quad \{\mathbf{v}\}_{\bar{x}} = \frac{\mu}{h}\begin{Bmatrix}-\sin\theta\\ e+\cos\theta\\ 0\end{Bmatrix}$$
2. Calculate matrix Q (e.g. Euler zxz) $[\mathbf{Q}]_{\bar{x}\bar{X}} =$ $$\begin{bmatrix}\cos\omega & \sin\omega & 0\\ -\sin\omega & \cos\omega & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 0 & \cos i & \sin i\\ 0 & -\sin i & \cos i\end{bmatrix}\begin{bmatrix}\cos\Omega & \sin\Omega & 0\\ -\sin\Omega & \cos\Omega & 0\\ 0 & 0 & 1\end{bmatrix}$$
3. Re-calculate r, v in ECI frame $\{\mathbf{r}\}_X = [\mathbf{Q}]_{\bar{x}X}\{\mathbf{r}\}_{\bar{x}}$ $\{\mathbf{v}\}_X = [\mathbf{Q}]_{\bar{x}X}\{\mathbf{v}\}_{\bar{x}}$

Rigid Body Dynamics

Rotating Body time derivatives of vector (orbit) Given that $\vec{\omega} = \frac{d\theta}{dt}$ and $\vec{\alpha} = \frac{d\vec{\omega}}{dt}$

$\frac{d\vec{A}}{dt} = \vec{\omega}\times\vec{A}$ and $\frac{d^2\vec{A}}{dt^2} = \vec{\alpha}\times\vec{A} + \vec{\omega}\times(\vec{\omega}\times\vec{A})$

Time derivative of moving vector in body frame Given that $\vec{B} = [B_x\hat{I} + B_y\hat{J} + B_z\hat{K}](inertial) = [b_x\hat{j} + b_y\hat{j} + b_z\hat{k}](rotatingframe)$

We have $\frac{d\vec{B}}{dt} = \frac{db_x}{dt}\hat{i} + \frac{db_y}{dt}\hat{j} + \frac{db_z}{dt}\hat{k} + \vec{\omega}\times\vec{B}$ i.e. the time derivative of $\vec{B}$ in inertial frame is equal to the time derivative of $\vec{B}$ in the rotating frame, plus the correction term $\vec{\omega}\times\vec{B}$.